Bosnia Herzegovina Team Selection Test 2017

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Day 1 May 13th

1 Incircle of triangle ABC touches AB, AC at P, Q. BI, CI intersect with PQ at K, L. Prove that circumcircle of ILK is tangent to incircle of ABC if and only if $AB + AC = 3BC$.

2 Denote by $\mathbb{N}$ the set of all positive integers. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all positive integers m and n, the integer $f(m)+f(n)-mn$ is nonzero and divides $mf(m)+nf(n)$.

Proposed by Dorlir Ahmeti, Albania

3 Find all real constants c for which there exist strictly increasing sequence a of positive integers such that $(a_{2n-1} + a_{2n})/a_n = c$ for all positive intgers n.

Day 2 May 14th

4 There are $6n+4$ mathematicians participating in a conference which includes $2n+1$ meetings. Each meeting has one round table that suits for 4 people and n round tables that each table suits for 6 people. We have known that two arbitrary people sit next to or have opposite places doesn't exceed one time.
1. Determine whether or not there is the case $n = 1$.
2. Determine whether or not there is the case $n > 1$.

5 Find the smallest constant $C > 0$ for which the following statement holds: among any five positive real numbers a_1, a_2, a_3, a_4, a_5 (not necessarily distinct), one can always choose distinct subscripts i, j, k, l such that

$$\left| \frac{a_i}{a_j} - \frac{a_k}{a_l} \right| \leq C.$$

6 Given is an acute triangle ABC. M is an arbitrary point at the side AB and N is midpoint of AC. The foots of the perpendiculars from A to MC and MN are points P and Q. Prove that center of the circumcircle of triangle PQN lies on the fixed line for all points M from the side AB.