Balkan MO 2014

– May 4th

1 Let x, y and z be positive real numbers such that $xy + yz + xz = 3xyz$. Prove that

$$x^2y + y^2z + z^2x \geq 2(x + y + z) - 3$$

and determine when equality holds.

UK - David Monk

2 A *special number* is a positive integer n for which there exists positive integers a, b, c, and d with

$$n = \frac{a^3 + 2b^3}{c^3 + 2d^3}.$$

Prove that

i) there are infinitely many special numbers;
ii) 2014 is not a special number.

Romania

3 Let $ABCD$ be a trapezium inscribed in a circle Γ with diameter AB. Let E be the intersection point of the diagonals AC and BD . The circle with center B and radius BE meets Γ at the points K and L (where K is on the same side of AB as C). The line perpendicular to BD at E intersects CD at M. Prove that KM is perpendicular to DL.

Greece - Silouanos Brazitikos

4 Let n be a positive integer. A regular hexagon with side length n is divided into equilateral triangles with side length 1 by lines parallel to its sides.
Find the number of regular hexagons all of whose vertices are among the vertices of those equilateral triangles.

UK - Sahl Khan