26-th Balkan Mathematical Olympiad

Kragujevac, Serbia – May 2, 2009

1. Solve the equatio
$$3^x - 5^y = z^2$$
in positive integers.

2. Let MN be a line parallel to the side BC of triangle ABC, with M on the side AB and N on the side AC. The lines BN and CM meet at point P. The circumcircles of triangles BMP and CNP meet at two distinct points P and Q. Prove that $\angle BAQ = \angle CAP$.

3. A 9×12 rectangle is partitioned into unit squares. The centers of all the unit squares, except for the four corner squares and the eight squares sharing a common side with one fo them, are colored red. Is it possible to label these red centers $C_1, C_2, \ldots, C_{96}$ in such a way that the following two conditions are both fulfilled:

   (i) the distances $C_1C_2, C_2C_3, \ldots, C_{95}C_{96}, C_{96}C_1$ are all equal to $\sqrt{13}$,

   (ii) the closed broken line $C_1C_2 \ldots C_{96}C_1$ has a center of symmetry?

4. Denote by S the set of all positive integers. Find all functions $f : S \to S$ such that
$$f(f(m)^2 + 2f(n)^2) = m^2 + 2n^2, \quad \text{for all } m, n \in S.$$

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